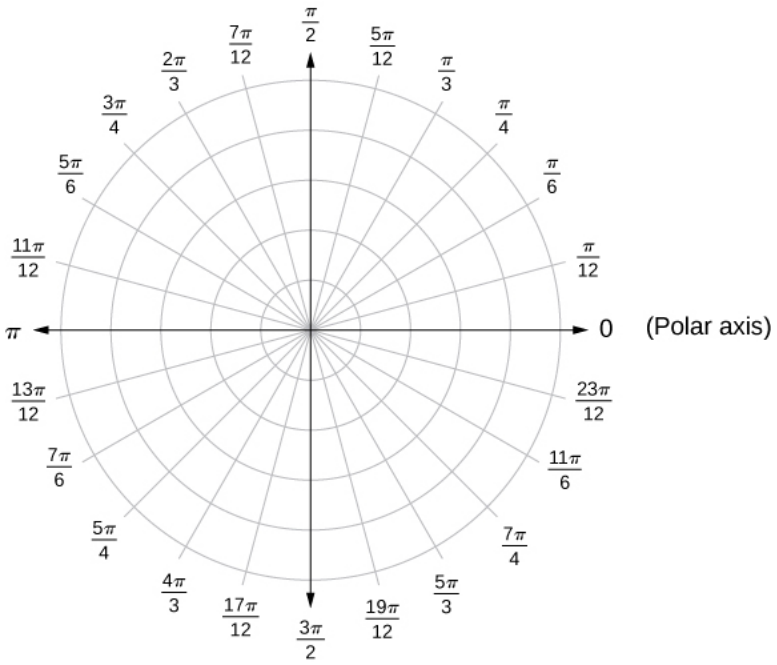
MATH 147 Review: Arithmetic for Complex Numbers

1. Plot $1 + \sqrt{3}i$.

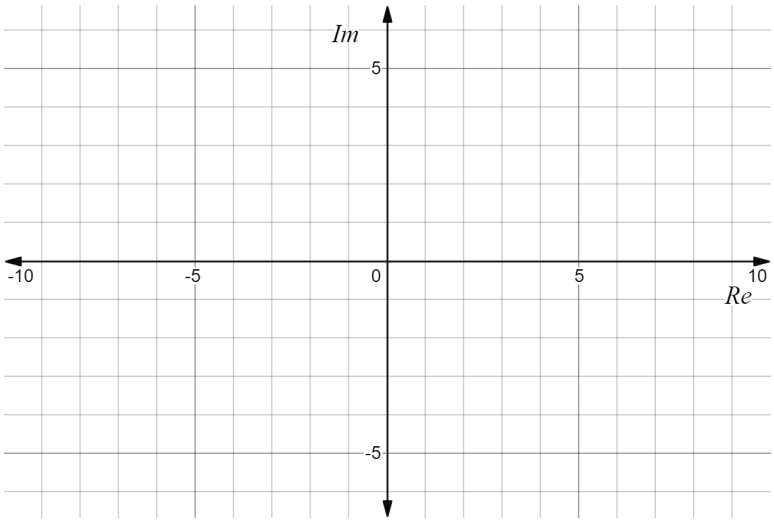


Solution:

$$1 + \sqrt{3}i = 2e^{i\frac{\pi}{3}}$$

The radial length is 2 and the polar angle is $\frac{\pi}{3}$.

2. Plot $2e^{i\frac{\pi}{6}}$.



Solution:

$$2e^{i\frac{\pi}{6}} = \sqrt{3} + i$$

The horizontal length is $\sqrt{3} \approx 1.7$ and the vertical length is 1.

3. Find $z + w$, $z \cdot w$, $\frac{1}{z}$ in both rectangular and polar form where $z = \sqrt{3} + i$ and $w = 1 + \sqrt{3}i$.

Solution:

$$\begin{aligned}
z + w &= (\sqrt{3} + i) + (1 + \sqrt{3}i) \\
&= (1 + \sqrt{3}) + (1 + \sqrt{3})i = (1 + \sqrt{3})\sqrt{2}e^{i\frac{\pi}{4}} \\
z \cdot w &= (\sqrt{3} + i)(1 + \sqrt{3}i) \\
&= 4i = 4e^{i\frac{\pi}{2}} \\
\frac{1}{z} &= \frac{1}{\sqrt{3} + i} \\
&= \frac{\sqrt{3} - i}{4} \\
&= \frac{\sqrt{3}}{4} - \frac{1}{4}i \\
\frac{1}{z} &= \frac{1}{2e^{i\frac{\pi}{6}}} \\
&= \frac{1}{2}e^{i(-\frac{\pi}{6})}
\end{aligned}$$